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Derivatives Formulas involving I-function of two variables and generalized M-series

Y. Pragathi Kumar[1], Gebreegziabher Hailu[2], Alem Mabrahtu[3], B. Satyanarayana[4]

Department of Mathematics, College of Natural and Computational Sciences, Adigrat University, Adigrat, Ethiopia [1,2,3]

Department of Mathematics, Acharya Nagarjuna University, Nagarjuna Nagar, India[4]

Abstract: The objective of this paper is to establish some derivative formulae of I-function of two variables involving generalized M-series. The special cases of our derivatives yield interesting results.

Keywords: I-function, Mellin-Barnes contour integral, generalized M-series.

1. INTRODUCTION

Recently, Satyanarayana et al. [9, 10] are obtained some differentiation formulae for I-function of two variables with general class of polynomials and Struve's function. In the present paper we establish derivative formulae of I-function of two variables involving Generalized M-series.
We shall utilize the following formulae and notations in the present investigation. The I-function of two variables defined by Shantha Kumari et al.[13] (and also see Satyanarayana et al. [12]).

(1.1) $I[z_1, z_2] =$

$$I^{0,n_1:m_2,n_2;m_3,n_3}_{p_1,q_1:p_2,q_2;p_3,q_3}\left[\begin{matrix} z_1 \\ z_2 \end{matrix}\middle|\begin{matrix} (a_j;\alpha_j,A_j;\xi_j)_{1,p_1} : (c_j,C_j;U_j)_{1,p_2};(e_j,E_j;P_j)_{1,p_3} \\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1} : (d_j,D_j;V_j)_{1,q_2};(f_j,F_j;Q_j)_{1,q_3} \end{matrix}\right]$$

$$= \frac{1}{(2\pi i)^2}\int_{L_s}\int_{L_t} \phi(s,t)\,\theta_1(s)\,\theta_2(t)\, z_1^{s} z_2^{t}\, ds\, dt$$

where

$$\phi(s,t) = \frac{\prod_{j=1}^{n_1}\Gamma^{\xi_j}\left(1-a_j+\alpha_j s+A_j t\right)}{\prod_{j=n_1+1}^{p_1}\Gamma^{\xi_j}\left(a_j-\alpha_j s-A_j t\right)\prod_{j=1}^{q_1}\Gamma^{\eta_j}\left(1-b_j+\beta_j s+B_j t\right)}$$

$$\theta_1(s) = \frac{\prod_{j=1}^{n_2}\Gamma^{U_j}(1-c_j+C_j s)\prod_{j=1}^{m_2}\Gamma^{V_j}(d_j-D_j s)}{\prod_{j=n_2+1}^{p_2}\Gamma^{U_j}(c_j-C_j s)\prod_{j=m_2+1}^{q_2}\Gamma^{V_j}(1-d_j+D_j s)}$$

$$\theta_2(t) = \frac{\prod_{j=1}^{n_3}\Gamma^{P_j}(1-e_j+E_j t)\prod_{j=1}^{m_3}\Gamma^{Q_j}(f_j-F_j t)}{\prod_{j=n_3+1}^{p_3}\Gamma^{P_j}(e_j-E_j t)\prod_{j=m_3+1}^{q_3}\Gamma^{Q_j}(1-f_j+F_j t)}$$

where $n_j, p_j, q_j (j = 1, 2, 3)$, $m_j (j = 2, 3)$ are non negative integers such that $0 \le n_j \le p_j$, $q_1 \ge 0$, $0 \le m_j \le q_j (j = 2, 3)$ (not all zero simultaneously). $\alpha_j, A_j (j = 1, \ldots, p_1)$; $\beta_j, B_j (j = 1, \ldots, q_1)$, $C_j (j = 1, \ldots, p_2)$, $D_j (j = 1, \ldots, q_2)$, $E_j (j = 1, \ldots, p_3)$, $F_j (j = 1, \ldots, q_3)$ are positive quantities. $a_j (j = 1, \ldots, p_1)$, $b_j (j = 1, \ldots, q_1)$, $c_j (j = 1, \ldots, p_2)$, $d_j (j = 1, \ldots, q_2)$, $e_j (j = 1, \ldots, p_3)$ and $f_j (j = 1, \ldots, q_3)$ are complex numbers. The exponents $\xi_j, \eta_j, U_j, V_j, P_j, Q_j$ may take non integer values.

L_s and L_t are suitable contours of Mellin-Barnes type. More over, the contour L_s is in the complex s-plane and runs from $\sigma_1 - i\infty$ to $\sigma_1 + i\infty$ (σ_1 real), so that all the poles of $\Gamma^{V_j}(d_j - D_j s)$ $(j = 1, \ldots, m_2)$ lie to the right of L_s and all poles of $\Gamma^{U_j}(1-c_j+C_j s)$ $(j = 1, \ldots, n_2)$, $\Gamma^{\xi_j}\left(1-a_j+\alpha_j s+A_j t\right)$ $(j = 1, \ldots, n_1)$ lie to the left of L_s. Similar conditions for L_t follows in complex t-plane. The detailed conditions of this function can be found in Shantha Kumari et al.[13].

The Generalized M-Series is defined by Sharma and Renu[16] as

(1.2) $$\underset{p\ q}{\overset{\alpha,\beta}{M}} = \underset{p\ q}{\overset{\alpha,\beta}{M}}(a_1,\ldots,a_p;b_1,\ldots,b_q;z)$$

$$= \sum_{k=0}^{\infty}\frac{(a_1)_k\ldots(a_p)_k}{(b_1)_k\ldots(b_q)_k}\frac{z^k}{\Gamma(\alpha k+\beta)},\ z,\alpha,\beta \in \mathbb{C},$$

$\Re(\alpha) > 0$.

Series is convergent for all z if q ≥ p, it is convergent for |z|<1 if p = q+1 and divergent if p > q+1. Where p = q+1 and |z| = 1, the series convergent in some case. Let $\beta = \sum_{j=1}^{p} a_j - \sum_{j=1}^{q} b_j$

It can be shown that when p = q+1 the series is absolutely convergent for |z|=1 if $\Re(\beta)<0$. Conditionally convergent for z = -1 if $0 \le \Re(\beta) \le 1$ and divergent for |z|=1 if $1 \le \Re(\beta)$.

$$(1.3)\quad D_x = \frac{d}{dx}$$

$$(1.4)\quad D_x^r f(x) = \frac{d^r}{dx^r} f(x)$$

$$(1.5)\quad (xD_x)^r f(x) = \left(x\frac{d}{dx}\right)^r f(x)$$

$$(1.6)\quad (D_x x)^r f(x) = \left(\frac{d}{dx}x\right)^r f(x)$$

2. MAIN RESULTS

In this section, we establish some derivative formulae of I-function of two variables involving generalized M-series.

Theorem 1. Prove that

(2.1)

$$D_x^r\left\{ \overset{\alpha,\beta}{\underset{l\ \ m}{M}} (a_1,....,a_l;b_1,....,b_m;ax^\lambda) I[z_1x^{h_1}, z_2x^{h_2}]\right\}$$

$$= \sum_{k=0}^{\infty} \frac{(a_1)_k....(a_l)_k}{(b_1)_k....(b_m)_k} \frac{a^k}{\Gamma(\alpha k+\beta)} x^{\lambda k - r}$$

$$I^{0,n_1+1:m_2,n_2;m_3,n_3}_{p_1,q_1+1:p_2,q_2;p_3,q_3}\left[\begin{matrix} z_1x^{h_1} \\ z_2x^{h_2}\end{matrix}\middle| \begin{matrix}(-\lambda k;h_1,h_2;1),(a_j;\alpha_j,A_j;\xi_j)_{1,p_1}: (c_j,C_j;U_j)_{1,p_2};(e_j,E_j;P_j)_{1,p_3} \\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1},(r-\lambda k;h_1,h_2;1): (d_j,D_j;V_j)_{1,q_2};(f_j,F_j;Q_j)_{1,q_3}\end{matrix}\right]$$

Where z,α, β and λ are complex numbers and h_1, h_2 are real and positive.

Proof. To prove this theorem, we consider

$$D_x^r\left\{ \overset{\alpha,\beta}{\underset{l\ \ m}{M}} (a_1,....,a_l;b_1,....,b_m;ax^\lambda) I[z_1x^{h_1}, z_2x^{h_2}]\right\}$$

Expressing I-function of two variables as contour integral (1.1), generalized M-series (1.2) and evaluating derivatives with help of the notation (1.5), we get

$$(2.4)\quad D_x^r\left\{ \overset{\alpha,\beta}{\underset{l\ \ m}{M}} (a_1,....,a_l;b_1,....,b_m;ax^\lambda) I[z_1x^{h_1}, z_2x^{h_2}]\right\}$$

$$= \sum_{k=0}^{\infty} \frac{(a_1)_k....(a_l)_k}{(b_1)_k....(b_m)_k} \frac{a^k}{\Gamma(\alpha k+\beta)}$$

$$\frac{1}{(2\pi i)^2}\int_{L_s}\int_{L_t}\Big\{\varphi(s,t)\theta_1(s)\theta_2(t)z_1^s z_2^t \times \prod_{j=0}^{r-1}(\lambda k + h_1 s + h_2 t - j) x^{\lambda k + h_1 s + h_2 t - r}\Big\} ds\, dt$$

Now it is easily express that [13]

$$(2.5)\quad \prod_{j=0}^{r-1}(\lambda k + h_1 s + h_2 t - j) = \frac{\Gamma(1+\lambda k + h_1 s + h_2 t)}{\Gamma(1+\lambda k + h_1 s + h_2 t - r)}$$

Substitute (2.5) in (2.4), we get required result.

Theorem 2. Prove that

$$(2.6)\quad (xD_x - k_1)(xD_x - k_2).....(xD_x - k_r)$$

$$\left\{ \overset{\alpha,\beta}{\underset{l\ \ m}{M}} (a_1,....,a_l;b_1,....,b_m;ax^\lambda) I[z_1x^{h_1}, z_2x^{h_2}]\right\} =$$

$$= \sum_{k=0}^{\infty} \frac{(a_1)_k....(a_l)_k}{(b_1)_k....(b_m)_k} \frac{\left(ax^\lambda\right)^k}{\Gamma(\alpha k+\beta)}$$

$$I^{0,n_1+r:m_2,n_2;m_3,n_3}_{p_1,q_1+r:p_2,q_2;p_3,q_3}\left[\begin{matrix} z_1x^{h_1} \\ z_2x^{h_2}\end{matrix}\middle| \begin{matrix}(k_j-\lambda k;h_1,h_2;1)_{1,r},(a_j;\alpha_j,A_j;\xi_j)_{1,p_1}:(c_j,C_j;U_j)_{1,p_2};(e_j,E_j;P_j)_{1,p_3} \\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1},(1+k_j-\lambda k;h_1,h_2;1)_{1,r}:(d_j,D_j;V_j)_{1,q_2};(f_j,F_j;Q_j)_{1,q_3}\end{matrix}\right]$$

Where z,α, β and λ are complex numbers and h_1, h_2 are real and positive.

Proof. To prove this theorem, we consider

$$(xD_x - k_1)(xD_x - k_2)....(xD_x - k_r)$$

$$\left\{ \overset{\alpha,\beta}{\underset{l\ \ m}{M}} (a_1,....,a_l;b_1,....,b_m;ax^\lambda) I[z_1x^{h_1}, z_2x^{h_2}]\right\}$$

expressing I-function of two variables as contour integral (1.1), generalized M-series (1.2) and evaluating derivatives with help of the notation (1.5), we obtain

$$(2.7)\quad (xD_x - k_1)(xD_x - k_2)....(xD_x - k_r)$$

$$\left\{ \overset{\alpha,\beta}{\underset{l\ \ m}{M}} (a_1,....,a_l;b_1,....,b_m;ax^\lambda) I[z_1x^{h_1}, z_2x^{h_2}]\right\}$$

$$= \sum_{k=0}^{\infty} \frac{(a_1)_k....(a_l)_k}{(b_1)_k....(b_m)_k} \frac{a^k}{\Gamma(\alpha k+\beta)} \frac{1}{(2\pi i)^2}$$

$$\int_{L_s}\int_{L_t}\Big\{\varphi(s,t)\theta_1(s)\theta_2(t)z_1^s z_2^t \times \prod_{j=1}^{r}(\lambda k - k_j + h_1 s + h_2 t) x^{\lambda k + h_1 s + h_2 t}\Big\} ds\, dt$$

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By writing [13]

(2.8) $$\prod_{j=1}^{r}\left(\lambda k-k_j+h_1 s+h_2 t\right)=\prod_{j=1}^{r}\frac{\Gamma\left(1+\lambda k-k_j+h_1 s+h_2 t\right)}{\Gamma\left(\lambda k-k_j+h_1 s+h_2 t\right)}$$

in (2.7) and by using (1.1) we get equation (2.6).

Theorem 3. Prove that

(2.9) $$(D_x x-k_1)(D_x x-k_2).....(D_x x-k_r)\left\{\underset{l\ \ m}{\overset{\alpha,\beta}{M}}(a_1,....,a_l;b_1,....,b_m;ax^{\lambda})I[z_1x^{h_1},z_2x^{h_2}]\right\}$$

$$=\sum_{k=0}^{\infty}\frac{(a_1)_k....(a_l)_k}{(b_1)_k....(b_m)_k}\frac{\left(ax^{\lambda}\right)^k}{\Gamma(\alpha k+\beta)}$$

$$I^{0,n_1+r:m_2,n_2;m_3,n_3}_{p_1,q_1+r:p_2,q_2;p_3,q_3}\left[\begin{matrix}z_1x^{h_1}\\ z_2x^{h_2}\end{matrix}\middle|\begin{matrix}(k_j-\lambda k-1;h_1,h_2;1)_{1,r},(a_j;\alpha_j,A_j;\xi_j)_{1,p_1}:(c_j,C_j;U_j)_{1,p_2};(e_j,E_j;P_j)_{1,p_3}\\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1},(k_j-\lambda k;h_1,h_2;1)_{1,r}:(d_j,D_j;V_j)_{1,q_2};(f_j,F_j;Q_j)_{1,q_3}\end{matrix}\right]$$

Where z,α,β and λ are complex numbers and h_1, h_2 are real and positive.

Proof. Proof is similar as proof of theorem 1 and 2.

3. SPECIAL CASES

(i) By writing k1= k_2= …. = k_r= 0 in (2.6), we get (3.1)

$$(xD_x)^r\left\{\underset{l\ \ m}{\overset{\alpha,\beta}{M}}(a_1,....,a_l;b_1,....,b_m;ax^{\lambda})I[z_1x^{h_1},z_2x^{h_2}]\right\}$$

$$=\sum_{k=0}^{\infty}\frac{(a_1)_k....(a_l)_k}{(b_1)_k....(b_m)_k}\frac{\left(ax^{\lambda}\right)^k}{\Gamma(\alpha k+\beta)}$$

$$I^{0,n_1+r:m_2,n_2;m_3,n_3}_{p_1,q_1+r:p_2,q_2;p_3,q_3}\left[\begin{matrix}z_1x^{h_1}\\ z_2x^{h_2}\end{matrix}\middle|\begin{matrix}(-\lambda k;h_1,h_2;1)_{1,r},(a_j;\alpha_j,A_j;\xi_j)_{1,p_1}:(c_j,C_j;U_j)_{1,p_2};(e_j,E_j;P_j)_{1,p_3}\\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1},(1-\lambda k;h_1,h_2;1)_{1,r}:(d_j,D_j;V_j)_{1,q_2};(f_j,F_j;Q_j)_{1,q_3}\end{matrix}\right]$$

(ii) By taking k1= k_2= ….. = k_r= 0 in (2.9), we get(3.2)

$$(D_x x)^r\left\{\underset{l\ \ m}{\overset{\alpha,\beta}{M}}(a_1,....,a_l;b_1,....,b_m;ax^{\lambda})I[z_1x^{h_1},z_2x^{h_2}]\right\}$$

$$=\sum_{k=0}^{\infty}\frac{(a_1)_k....(a_l)_k}{(b_1)_k....(b_m)_k}\frac{\left(ax^{\lambda}\right)^k}{\Gamma(\alpha k+\beta)}$$

$$I^{0,n_1+r:m_2,n_2;m_3,n_3}_{p_1,q_1+r:p_2,q_2;p_3,q_3}\left[\begin{matrix}z_1x^{h_1}\\ z_2x^{h_2}\end{matrix}\middle|\begin{matrix}(-\lambda k-1;h_1,h_2;1)_{1,r},(a_j;\alpha_j,A_j;\xi_j)_{1,p_1}:(c_j,C_j;U_j)_{1,p_2};(e_j,E_j;P_j)_{1,p_3}\\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1},(-\lambda k;h_1,h_2;1)_{1,r}:(d_j,D_j;V_j)_{1,q_2};(f_j,F_j;Q_j)_{1,q_3}\end{matrix}\right]$$

(iii) For β = 1in (2.1),(2.6) and (2.9), we get derivative formulae involving I-function of two variables involving Generalized M-series by Sharma [15], respectively

(3.3)

$$D_x^r\left\{\underset{l\ \ m}{\overset{\alpha}{M}}(a_1,....,a_l;b_1,....,b_m;ax^{\lambda})I[z_1x^{h_1},z_2x^{h_2}]\right\}$$

$$=\sum_{k=0}^{\infty}\frac{(a_1)_k....(a_l)_k}{(b_1)_k....(b_m)_k}\frac{a^k}{\Gamma(\alpha k+1)}x^{\lambda k-r}$$

$$I^{0,n_1+1:m_2,n_2;m_3,n_3}_{p_1,q_1+1:p_2,q_2;p_3,q_3}\left[\begin{matrix}z_1x^{h_1}\\ z_2x^{h_2}\end{matrix}\middle|\begin{matrix}(-\lambda k;h_1,h_2;1),(a_j;\alpha_j,A_j;\xi_j)_{1,p_1}:(c_j,C_j;U_j)_{1,p_2};(e_j,E_j;P_j)_{1,p_3}\\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1},(r-\lambda k;h_1,h_2;1):(d_j,D_j;V_j)_{1,q_2};(f_j,F_j;Q_j)_{1,q_3}\end{matrix}\right]$$

(3.4) $$(xD_x-k_1)(xD_x-k_2).....(xD_x-k_r)\left\{\underset{l\ \ m}{\overset{\alpha}{M}}(a_1,....,a_l;b_1,....,b_m;ax^{\lambda})I[z_1x^{h_1},z_2x^{h_2}]\right\}$$

$$=\sum_{k=0}^{\infty}\frac{(a_1)_k....(a_l)_k}{(b_1)_k....(b_m)_k}\frac{\left(ax^{\lambda}\right)^k}{\Gamma(\alpha k+1)}$$

$$I^{0,n_1+r:m_2,n_2;m_3,n_3}_{p_1,q_1+r:p_2,q_2;p_3,q_3}\left[\begin{matrix}z_1x^{h_1}\\ z_2x^{h_2}\end{matrix}\middle|\begin{matrix}(k_j-\lambda k;h_1,h_2;1)_{1,r},(a_j;\alpha_j,A_j;\xi_j)_{1,p_1}:(c_j,C_j;U_j)_{1,p_2};(e_j,E_j;P_j)_{1,p_3}\\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1},(1+k_j-\lambda k;h_1,h_2;1)_{1,r}:(d_j,D_j;V_j)_{1,q_2};(f_j,F_j;Q_j)_{1,q_3}\end{matrix}\right]$$

(3.5) $$(D_x x-k_1)(D_x x-k_2).....(D_x x-k_r)\left\{\underset{l\ \ m}{\overset{\alpha}{M}}(a_1,....,a_l;b_1,....,b_m;ax^{\lambda})I[z_1x^{h_1},z_2x^{h_2}]\right\}$$

$$=\sum_{k=0}^{\infty}\frac{(a_1)_k....(a_l)_k}{(b_1)_k....(b_m)_k}\frac{\left(ax^{\lambda}\right)^k}{\Gamma(\alpha k+1)}$$

$$I^{0,n_1+r:m_2,n_2;m_3,n_3}_{p_1,q_1+r:p_2,q_2;p_3,q_3}\left[\begin{matrix}z_1x^{h_1}\\ z_2x^{h_2}\end{matrix}\middle|\begin{matrix}(k_j-\lambda k-1;h_1,h_2;1)_{1,r},(a_j;\alpha_j,A_j;\xi_j)_{1,p_1}:(c_j,C_j;U_j)_{1,p_2};(e_j,E_j;P_j)_{1,p_3}\\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1},(k_j-\lambda k;h_1,h_2;1)_{1,r}:(d_j,D_j;V_j)_{1,q_2};(f_j,F_j;Q_j)_{1,q_3}\end{matrix}\right]$$

(iv) By substituting l = m = 0 in (2.1),(2.6) and (2.9), we have derivative formulae of I-function of two variables involving Mittag-Leffler function respectively

(3.6) $$D_x^r\left\{E_{\alpha,\beta}(ax^{\lambda})I[z_1x^{h_1},z_2x^{h_2}]\right\}$$

$$=\sum_{k=0}^{\infty}\frac{a^k}{\Gamma(\alpha k+\beta)}x^{\lambda k-r}$$

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$$I^{0,n_1+1:m_2,n_2;m_3,n_3}_{p_1,q_1+1:p_2,q_2;p_3,q_3}\left[\begin{matrix} z_1x^{h_1} \\ z_2x^{h_2}\end{matrix}\middle| \begin{matrix}(-\lambda k;h_1,h_2;1),\ (a_j;\alpha_j,A_j;\xi_j)_{1,p_1}:(c_j,C_j;U_j)_{1,p_2};(e_j,E_j;P_j)_{1,p_3} \\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1},\ (r-\lambda k;h_1,h_2;1):\ (d_j,D_j;V_j)_{1,q_2};(f_j,F_j;Q_j)_{1,q_3}\end{matrix}\right]$$

(3.7) $(xD_x-k_1)(xD_x-k_2)\ldots(xD_x-k_r)\left\{E_{\alpha,\beta}(ax^{\lambda})I[z_1x^{h_1},z_2x^{h_2}]\right\}$

$$=\sum_{k=0}^{\infty}\frac{\left(ax^{\lambda}\right)^k}{\Gamma(\alpha k+\beta)}\, I^{0,n_1+r:m_2,n_2;m_3,n_3}_{p_1,q_1+r:p_2,q_2;p_3,q_3}\left[\begin{matrix} z_1x^{h_1} \\ z_2x^{h_2}\end{matrix}\middle| \begin{matrix}(k_j-\lambda k;h_1,h_2;1)_{1,r},\ (a_j;\alpha_j,A_j;\xi_j)_{1,p_1}:\ (c_j,C_j;U_j)_{1,p_2};(e_j,E_j;P_j)_{1,p_3} \\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1},\ (1+k_j-\lambda k;h_1,h_2;1)_{1,r}:(d_j,D_j;V_j)_{1,q_2};(f_j,F_j;Q_j)_{1,q_3}\end{matrix}\right]$$

(3.8) $(D_x x-k_1)(D_x x-k_2)\ldots(D_x x-k_r)\left\{E_{\alpha,\beta}(ax^{\lambda})I[z_1x^{h_1},z_2x^{h_2}]\right\}$

$$=\sum_{k=0}^{\infty}\frac{\left(ax^{\lambda}\right)^k}{\Gamma(\alpha k+\beta)}\, I^{0,n_1+r:m_2,n_2;m_3,n_3}_{p_1,q_1+r:p_2,q_2;p_3,q_3}\left[\begin{matrix} z_1x^{h_1} \\ z_2x^{h_2}\end{matrix}\middle| \begin{matrix}(k_j-\lambda k-1;h_1,h_2;1)_{1,r},\ (a_j;\alpha_j,A_j;\xi_j)_{1,p_1}:\ (c_j,C_j;U_j)_{1,p_2};(e_j,E_j;P_j)_{1,p_3} \\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1},\ (k_j-\lambda k;h_1,h_2;1)_{1,r}:(d_j,D_j;V_j)_{1,q_2};(f_j,F_j;Q_j)_{1,q_3}\end{matrix}\right]$$

$E_{\alpha,\beta}(ax^{\lambda})$ is Mittag-Leffler function [6].

(v) Writing $\alpha=\beta=1$ in (2.1),(2.6) and (2.9), we obtain derivative formulae of I-function of two variables involving generalized hyper geometric function [5] respectively

(3.9) $D_x^r\left\{{}_lF_m(ax^{\lambda})I[z_1x^{h_1},z_2x^{h_2}]\right\}$

$$=\sum_{k=0}^{\infty}\frac{(a_1)_k\ldots(a_l)_k}{(b_1)_k\ldots(b_m)_k}\frac{a^k}{k!}x^{\lambda k-r}\, I^{0,n_1+1:m_2,n_2;m_3,n_3}_{p_1,q_1+1:p_2,q_2;p_3,q_3}\left[\begin{matrix} z_1x^{h_1} \\ z_2x^{h_2}\end{matrix}\middle| \begin{matrix}(-\lambda k;h_1,h_2;1),(a_j;\alpha_j,A_j;\xi_j)_{1,p_1}:\ (c_j,C_j;U_j)_{1,p_2};(e_j,E_j;P_j)_{1,p_3} \\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1},(r-\lambda k;h_1,h_2;1):\ (d_j,D_j;V_j)_{1,q_2};(f_j,F_j;Q_j)_{1,q_3}\end{matrix}\right]$$

(3.10) $(xD_x-k_1)(xD_x-k_2)\ldots(xD_x-k_r)\left\{{}_lF_m(ax^{\lambda})I[z_1x^{h_1},z_2x^{h_2}]\right\}$

$$=\sum_{k=0}^{\infty}\frac{(a_1)_k\ldots(a_l)_k}{(b_1)_k\ldots(b_m)_k}\frac{\left(ax^{\lambda}\right)^k}{k!}\, I^{0,n_1+r:m_2,n_2;m_3,n_3}_{p_1,q_1+r:p_2,q_2;p_3,q_3}\left[\begin{matrix} z_1x^{h_1} \\ z_2x^{h_2}\end{matrix}\middle| \begin{matrix}(k_j-\lambda k;h_1,h_2;1)_{1,r},\ (a_j;\alpha_j,A_j;\xi_j)_{1,p_1}:\ (c_j,C_j;U_j)_{1,p_2};(e_j,E_j;P_j)_{1,p_3} \\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1},\ (1+k_j-\lambda k;h_1,h_2;1)_{1,r}:(d_j,D_j;V_j)_{1,q_2};(f_j,F_j;Q_j)_{1,q_3}\end{matrix}\right]$$

(3.11) $(D_x x-k_1)(D_x x-k_2)\ldots(D_x x-k_r)\left\{{}_lF_m(ax^{\lambda})I[z_1x^{h_1},z_2x^{h_2}]\right\}$

$$=\sum_{k=0}^{\infty}\frac{(a_1)_k\ldots(a_l)_k}{(b_1)_k\ldots(b_m)_k}\frac{\left(ax^{\lambda}\right)^k}{k!}\, I^{0,n_1+r:m_2,n_2;m_3,n_3}_{p_1,q_1+r:p_2,q_2;p_3,q_3}\left[\begin{matrix} z_1x^{h_1} \\ z_2x^{h_2}\end{matrix}\middle| \begin{matrix}(k_j-\lambda k-1;h_1,h_2;1)_{1,r},\ (a_j;\alpha_j,A_j;\xi_j)_{1,p_1}:\ (c_j,C_j;U_j)_{1,p_2};(e_j,E_j;P_j)_{1,p_3} \\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1},\ (k_j-\lambda k;h_1,h_2;1)_{1,r}:(d_j,D_j;V_j)_{1,q_2};(f_j,F_j;Q_j)_{1,q_3}\end{matrix}\right].$$

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